Numerical prediction and sequential process optimization of sheet forming based on the genetic algorithm

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Abstract

The genetic algorithm is an emerging technique used in engineering design activities to find an optimized solution that satisfies a number of design goals. The non-linear direct method of goal search uses successive linearization techniques that are sensitive to the starting solution chosen and quality of the objective function. The proposed technique can solve programming problems with non-convex regions, which are usually avoided in classical optimization problems. The efficacy of the proposed method was demonstrated by solving a number of test problems. The results suggest that the proposed method is effective as a practical tool for solving sheet forming problems.

Keywords: Sheet forming optimization, genetic algorithms, goal programming

1. Introduction

It is important to investigate the numerical predictions and optimization schemes to determine how the blank is affected by varying the forming process and die parameters. The optimization objective in the forming process is achieved using genetic algorithms. These algorithms are known to be able to explore the entire functional space, and thus they can detect the global optimal solution. Incremental data are used to further limit the space that is investigated. The gradient of the field calculated for the simplex method eliminates candidates from the possible optimal solutions towards the global optimal solution and limits the scope of the data for the candidates obtained from generic algorithms.

2. Literature review

The finite element method is generally adopted for metal forming because it provides detailed information about the domain under study and is an essential component of computer-aided design. Kobayashi [1] applied a finite-element-based backward tracing technique to design an optimized pre-form. However, this technique is largely inefficient for determining the optimal solution due to the presence of diverse and multiple loading solution paths. Joun and Hwang [2] proposed a new approach for steady-state metal forming similar to the one examined here (extrusion) based on the iterative strategy using the gradient projection method, which is a valuable contribution to the methods of iterative optimization. In spite of the potential of their proposed method, the initial guess required by this method can influence the search, which is often stuck in a sub-optimal solution.

The optimization problem of finding the solution for the function $F(x)$ of the problem can be solved using a well-known quasi-Newton method proposed by physicist W.C. Davidson [3]. His idea is based on the Newton method, where it is assumed that a function can be locally approximated around the optimal solution, but that method requires that the optimized function be computed for each iteration. The quasi-Newton method uses a Hessian matrix and incremental analysis of the successive gradient vectors by imposing a simple constraint onto the Hessian estimate.

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The improved version, popularized by Fletcher and Powell and known as the method of Davidon-Fletcher-Powell (DFP) [3], which is based on the secant method, is no longer widely used. Among the most common quasi-Newton algorithms today is the Broyden-Fletcher-Goldfarb-Shenno (BFGS) [4] method, which uses the McCormick criteria. It does not hold up against the DFP method, and it can lead to difficulties because the search space is unknown when multiple diverse loading paths are present. In the global multi-minima search problem of convergence, it is necessary to identify particular lines of deformation or optimization lines. This problem can be solved using locally-minimizing curves, as presented in Himmelblau's work [5], which are considered in the weighted-parametric space, see [6]. However, it should be noted that the optimization goals of convergence in multi-objective optimization are much more difficult to reach than single-objective optimization. The damped oscillator equation known as heavy ball with friction (HBF) is a widely-used technique [7] to arrive at the global minimum, which supplements the aforementioned method. Direct search methods [8] remain an effective option and were revived in the context of modern computing because these methods are sometimes the only optimization methods available. Evolutionary computations [9] are another alternative, but they lack the preciseness of the direct methods. Jackiewicz [10,11,12] and Dimitriu [13] studied the behaviors of various metals at the macroscopic level using the evolutionary strategy and criteria, which are crucial to determine the parameters of the plastic deformation process. They studied ferrite, low-carbon steels, and aluminum alloys and identified the advantages of using the evolutionary strategy in these contexts. Experimental analysis contributes to the understanding of evolutionary dynamics and is crucial to the design of competent optimization techniques because the theoretical approach lacks the flexibility and practicality of experimental investigation. Thus, the theoretical treatment of algorithmic performance and verification of convergence and stability of particular algorithms can be investigated.

To address the limitations of the existing methods, a number of authors adopted genetic algorithms with embedded finite-element solvers to automate the search for the optimized solution. Roy [14] implemented an adaptive genetic algorithm for the shape optimization of a sheet forming process. This method can deliver good solutions; however, the GA-based approach using a finite element solver as embedded optimizer incurs severe computational costs because it requires a large number of solutions to converge. Articles [15,16] provide an overview of the methods of evolutionary multi-objective optimization applied to the material design and processes.

In this study, a novel adaptive search using the generic algorithm is proposed. To investigate the performance and potential of this novel strategy, we applied the direct method of descent based on the simplex procedure to the genetic functional space. The results show that our method determined the global minima in our test problems. The adaptive strategy proposed here may be applied to different problems related to forming optimization, such as sheet metal forming.

The remainder of this paper is organized as follows. Section 3 introduces the process optimization. Section 4 explains our approach to direct search, and Sections 5 and 6 present some experimental results. Section 7 discusses the conclusions and directions for future work.

3. Genetic algorithms in sequential process optimization

A review of the literature revealed that there is a growing interest in approaches to the sequential process optimization problems using genetic algorithms [1,17,18]. However, these attempts tend to consider the entire space of possible solutions as candidates for the optimal solution bounded by the criterion space.

Methods of sequential process optimization assume initial points of the optimal solution based on the knowledge of the physical problem and can be derived for the set of optimizing constraints. This process makes the optimization algorithms susceptible to the correctness of the selection, including the number of stages required to achieve the required convergence. In addition, this approach, while being exhaustive, requires significant computation time to obtain the optimal solution. It seems feasible to apply the gradient descending method to a non-dominated sorting genetic algorithm (NSGA) for the sheet forming problem with multiple stages based on the ideas from [17,19,20].

Moreover, the dependency between stages can be modeled based on the results obtained during the previous stage, such that the optimization algorithm evaluates alternatives before moving into the next stage.

Given a matrix describing a problem and Dirichlet boundary conditions on an optimizing domain Ω_D, the goal is to solve the problem

$$-\Delta u = F(x) \text{in } \Omega, u = u_0 \text{ on } \partial\Omega_D$$

where $F(x)$ is the function of the problem on a polygonal domain $\epsilon\Omega$, u is a solution on the domain Ω, and $u_0$is an initial solution on the domain Ω.

Let vector f_j^k denote a constraint of the objective function $F(x)$ and f be a continuous function. Then

$$F_i^k(x) = \sum_{j=1}^{n} f_{ij}^k(x) \ |i = 1,2, \dots p$$

where

$$f_j(x) = \{f_{ij}(x_{1j}, x_{2j}, \dots x_{mj}, x_{1k}, x_{2k}, \dots x_{rk})|, k = j-1, r < m\}$$

i is the ith objective, p is the number of objectives, m is the number of design variables at stage j,k is the constraint from the previous stage $j-1$, r is the number of constraints from the stage $j-1$, which is taken into consideration at stage j, and n is the number of stages; see Fig. 1.

Genetic String Representation Printing Area

x_{11}	x_{21}		x_{m1}	x_{12}	x_{22}		x_{m2}		x_{1n}	x_{2n}		x_{mn}
100…	101…	…	111…	110…	100…	…	111…	…	001…	010…	…	001…
stage 1				*stage 2*					*stage n*			

Fig. 1. String structure of a chromosome.

The weighed pairing algorithm [21] is used to select a chromosome to produce new offspring. The rank weighting is evaluated according to the following formula:

$$P_n = \frac{N_{keep} - n + 1}{\sum_{n=1}^{N_{keep}} n}$$

The process of mating and producing offspring is performed using chromosomes from two parents. A crossover point is selected stochastically between the first and last bits of the parents' chromosomes. Consequentially, the offspring contains binary codes of both parents.

Mutation Operator

A polynomial probability distribution is used to derive the next solution based on a parent solution. Let the parent solution be $y_i^{(j)}$. Then the following process applies to the procedure for obtaining each element for the next solution $y_i^{(j)+1}$. First, choose a random number u between 0 and 1. Next, calculate a parameter of mutation δ_q as follows:

$$\delta_q = \begin{cases} [2u + (1-2u)(1-\delta)^{\eta_m+1}]^{\frac{1}{\eta_m+1}} - 1, if\ u \leq 0.5; \\ 1 - [2(1-u) + 2(u-0.5)(1-\delta)^{\eta_m+1}]^{\frac{1}{\eta_m+1}}, otherwise \end{cases}$$

where $\delta = \min\left[\left(y_i^{(j)} - y_i^l\right), \left(u_i^u - y_i^{(j)}\right)\right] / \left(y_i^u - y_i^l\right)$. Here, parameter η_m is the distribution index for the mutation. Lastly, calculate the mutation as follows:

$$y_i^{(j)+1} - y_i^j + \delta_q\left(y_i^u - y_i^l\right).$$

The mutation probability p_m is based on the work of Gao [22], who evaluated it in the context of Markhov chain models. He showed that the probability of mutation and the smaller the population are inversely proportional to the GA convergence rate. In this case, it varied from $1/P$ until 1.0, which resulted in the most extensive parameter mutation occurring in the beginning and the least parameter mutation at the end of the simulation.

4. Algorithms of direct search

Methods for the direct search of an optimal solution are based on the evaluation of the objective function. These methods are usually based on empirical deductions and do not require a strong mathematical background. The characteristics of the convergence of direct methods and the speed of convergence are not well understood. However, these methods involve ideas similar to those in the methods of first and second order [8,23]. In certain cases, the effectiveness of the algorithms of direct search can be assessed in the context of certain classes of functions. Thus, it is possible to perform the assessment by comparing experimental data with the data obtained theoretically and to perform a comparative analysis of those results.

To achieve the optimization, the mathematical apparatus of the unconstrained method of Broyden–Fletcher–Goldfarb–Shanno (BFGS) was used for the solution based on the Newton equation:

$$B_i D_j = -gradF_i^k\left(x_j\right)$$

where B_i is an approximation to the Hessian matrix, j is a stage of the optimization, D_j is a search direction, $F_i^k\left(x_j\right)$ is the function at stage j, and -grad is the antigradient of the function $F_i^k\left(x_j\right)$.

The complete Hessian matrix is calculated at the first stage and is updated iteratively at each subsequent stage. Finally, the next point x_{j+1} is obtained based on a line search in the direction D_j.

In the first stage, the search direction P_k is calculated using the Eigen matrix of the order n. Note that in this case, $p_1 = w_1 = -gradf(x_0)$, and thus the first stage is calculated in a manner similar to the method of fastest descent. On the other hand, if $A_k = H - 1(x_k - 1)$ and $k_{sik} = 1$, to the problem becomes the classic Newton method.

Lemma 1
Let the objective function $f(x)$ be a convex function, and let there exist a Hessian matrix $H(x)$ for every $x, y \in \mathbb{R}^n$ such that

$$\|H(x)-H(y)\| \leq L|x-y|$$

Then, the Newton method converges as a quadratic function, and the following formula holds:

$$\left|x^{k}-x^{*}\right| \leq \frac{L}{\lambda_{1}}\left|x^{k-1}-x^{*}\right|^{2}, k \in \mathbb{N}$$

Proof

Given that λ_1 is equal to the minimal proper value of the Hessian matrix $H(x)$ of the objective function in the vicinity of x^*, which contains x^{k-1}, then $H(x)$is positive definite in this vicinity, and

$$\lambda_1|x|^2 \leq (H(x)x, x)$$

for each $x\epsilon$ vicinity of x^*.

The above concludes the optimization process at stage k. The Hessian matrix at stage j is computed as B_{j+1} by the addition of the two matrices:

$$B_{k+1} = B_k + U_k + V_k$$

where B_k is the Hessian matrix at stage k and $U_k$and V_k are the step-size matrices calculated at each stage.

Moreover, the convergence ratio of the highly convex objective function is described by a quadratic function. Therefore, the algorithm of the quasi-Newton method converges if, for each stage matrix, A_k is selected such that it approximates the matrix $H-1(x_k-1)$ at $x^{k-1}\epsilon\mathbb{R}^n$.The construction of the matrix $H-1(x_k-1)$ for matrix A_k given the gradient at x_{k-1} can be greatly simplified. This simplification can be achieved as follows. The sequence of the approximating matrices A_k is constructed according to

$$A_{k+1} = A_k + \Delta A_k, k\epsilon\mathbb{N}$$

where ΔA_k is the correction matrix of order n. Let $f(x) = \frac{1}{2}(Q_x, x) + (c, x)$ be a quadratic function with a positive definite matrix Q. In this case, function $f(x)$ is a highly convex function, and the following holds:

$$gradf(x) = Qx + c$$

$$\Delta\omega^k = gradf\left(x^{k-1}\right) - gradf\left(x^k\right) = Q\left(x^{k-1} - x^k\right)$$

Finally,

$$Q^{-1}\Delta\omega^k = -\Delta x^k, k \in \mathbb{N}$$

where matrix Q coincides with the Hessian matrix $H(x)$ of the function $f(x)$. QED.

5. Test Problem 1

A sheet metal part was drawn in the stamping machine using the developed strategy as an experimental test. A pressure P was applied to a blank located in the stamping machine through a die moving vertically with a speed S at a distance H, see Fig. 2. 2. This physical problem was converted to a goal problem by using two variables (speed V, stress P) as an idealization. The goals were to optimize the thickness variation Δ during the process and the stress P in the blank. The blank was drawn for 70 mm, and the initial thickness of the blank was taken as 1.0 mm.

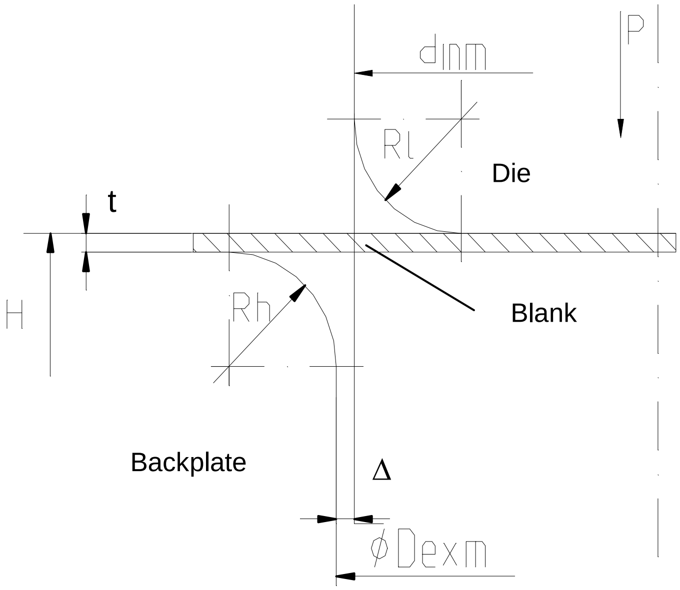

Fig. 2. Physical mode of the deep drawing process.

The following goal programming problem was used:

$$\text{Goal } (f_1(\Delta, P) = x_1 \le 230),$$
$$\text{Goal } (f_2(t) = x_2 < 1.06),$$
$$\text{Subject to } 95 \le x_1 \le 370,\ 1.0 \le x_2 \le 1.06.$$

The programming problem included two goals, which were both of the less-than-equal-to type. The first goal was to adhere to the physical definition of the problem and to stay within the limits of the stress-strain diagram of the given metal undergoing plastic deformation by building the association between the thickness and stress in the blank. The second goal complemented the first one, and it required that the thickness t not exceed the predefined thickness during the drawing process of the blank according to the specifications.

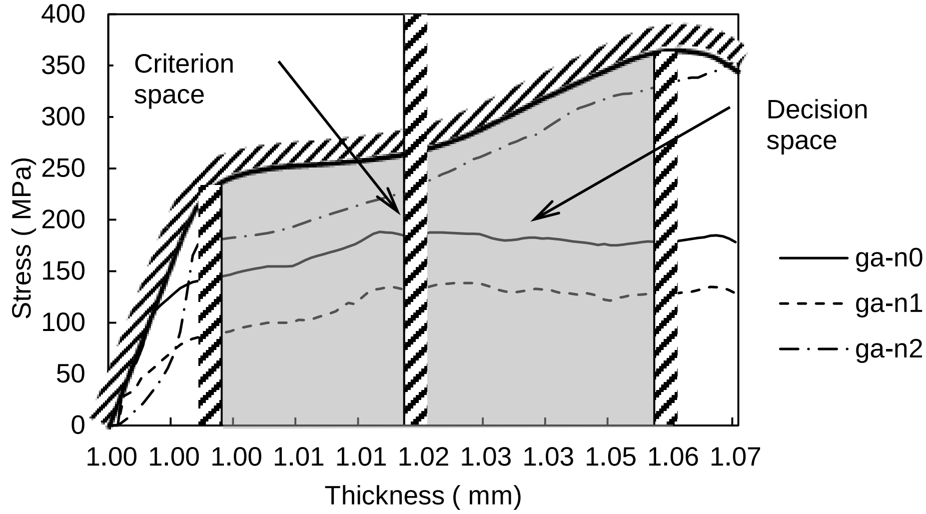

Fig. 3. The solution is shown in the problem space of thickness vs. stress dependency.

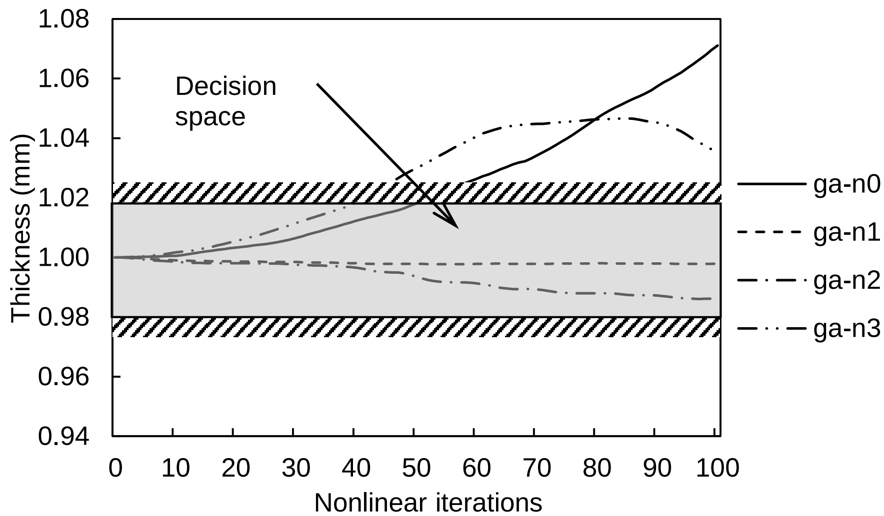

Fig. 4. The solution is shown in the problem space of increment vs. thickness dependency.

The feasible decision space is shown in Fig. 3, and the criterion space is the line between points A and B. Each discreet solution in this region corresponds to a goal programming problem for nodes ga-n0 through ga-n2. The solutions to this problem for Function 1 represent the plastic deformation of the body. The varying problem for Function 2 shown in Fig. 4 represents possible thicknesses of the deformable body undergoing plastic deformation, depending on the weight factors used for nodes ga-n0 through ga-n3. To solve this problem using NSGA, it was converted into an equivalent two-objective optimization problem as follows:

Minimize $\langle f_1(x_1) - 230 \rangle$,
Minimize $|f_2(x_2) - 1.06|$,
Subject to $95 \leq x_1 \leq 370, 1.0 \leq x_2 \leq 1.06$.

Fig. 3 and Fig. 4 show that NSGA found the potential optimal solutions, which converged to the global optimal solution.

6. Test Problem 2

The performance of the proposed method was evaluated, and an experiment was conducted to show how the process of drawing of the metal part in the stamping press can be optimized. The problem was to minimize the difference between the initial and optimal blanks and the thickness variation throughout the blank during the optimization of the sheet forming process. This multi-objective optimization problem was considered in a two-dimensional space using the Pareto front for the genetic algorithm optimization. The programming problem included two goals, where both were of the less-than-equal-to type, as follows:

Goal $(f_1(S,\mu) = x_1 < 0.15)$,
Goal $(f_2(S,t) = x_2 < 1.15)$,
Subject to $0 \leq x_1 \leq 0.15$, $1.0 \leq x_2 \leq 1.15$.

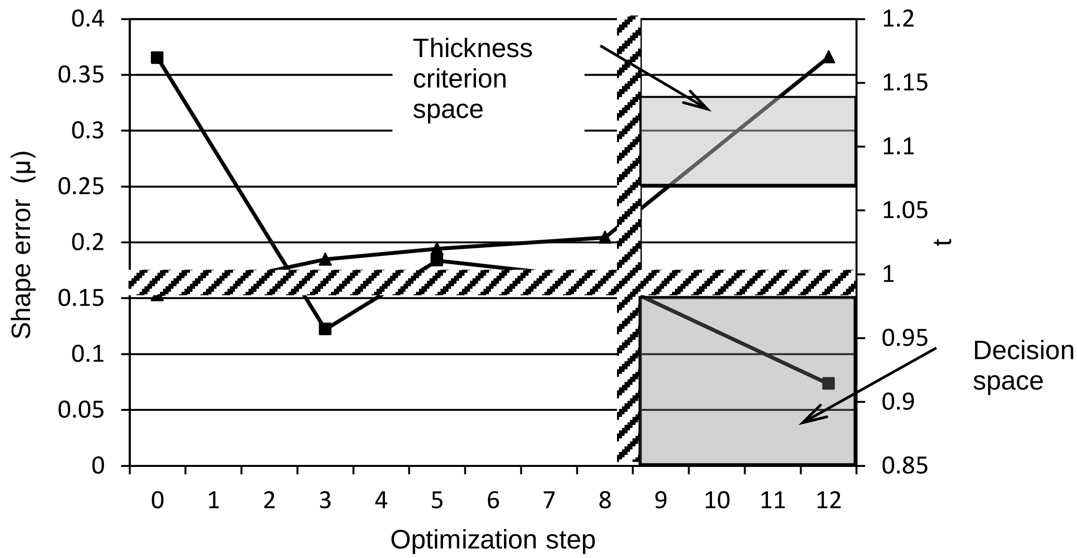

Fig. 5. The solution is shown in the problem space.

The first goal was to reach the final form of the blank and complete the drawing process according to the specifications. The second goal complemented the first one and was to minimize the thickness variation throughout the part during the process and to maintain the correlated error value within predefined constraints.

The primary y-axis in Fig. 6 shows the shape error μ_{error}, which is defined as the mean value of the shape difference between the current deformed contour of the flange and the target contour. The shape error was calculated by taking the arithmetic mean of the absolute values of the difference between the nodes on the current deformed contour and the nearest nodes on the target contour. The total value of the shape error can thus be expressed as follows:

$$\mu_{error} = \frac{1}{n}\sum_{i=1}^{n}|d_i|$$

where d_i is the difference between the nodes of the deformed contour on the current stage and the nearest nodes on the target contour and n is the total number of nodes located on the contour of the blank on the current stage. The thickness distribution is plotted for the profile of the cup on the secondary y-axis. The distribution shows that the model was effective, and the goal was to minimize this deviation of this parameter. The blank was drawn for 70 mm, and the initial thickness of the blank was taken as 1.0 mm.

$$\text{Minimize } \langle 0.15 - f_1(x_1) \rangle,$$
$$\text{Minimize } |f_2(x_2) - 1.0|,$$
$$\text{Subject to } 0 \le x_1 \le 0.15, 0.85 \le x_2 \le 1.15.$$

Because there existed no solution with an error μ_{error} greater than 0.15 and the thickness distribution was less than 0.02, the resulting solution was assumed to be the optimized solution. Fig. 6 shows that this solution converged to the following parameter values:

μ_{error}=0.07, $\quad$ $t = 1.0$

The solution for these parameters was the global optimal solution.

7. Conclusion

The sequential optimization of a sheet forming process was reviewed, and its limitations were addressed. It was shown that the generic algorithm was able to deliver a better solution compared to the traditional iterative approach based on the knowledge of the physical problem. Furthermore, the improved NSGA algorithm could be applied to this problem in anticipation of the optimal solution. The related portion genetic algorithms of creating a chromosome, obtaining a crossover and performing a mutation were mapped onto the problem space. The direct methods of global descent were used to improve the convergence speed. Finally, two test problems were presented to prove the validity of the ideas described in the paper. The results of these test problems indicated the feasibility of the proposed approach because their results converged nicely with the optimized solutions.

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